

Light neutrino contribution: is it all there is to neutrinoless double beta decay?

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We consider perturbative one loop QCD corrections to the light neutrino contribution to neutrinoless double beta decay and find large enhancement to the rate. QCD corrections also generate structures which mimic new physics contributions usually considered. Within some approximations, the net effect seem to almost saturate the experimental limits, and therefore seems to imply that this is all there is to neutrinoless double beta decay.

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The standard model (SM) of particle physics has been extremely successful in explaining almost all the observations, and the places where it has not been successful often have been marred with theoretical uncertainties. However, the experimental confirmation of the fact that neutrinos, which are strictly massless within SM, are massive particles (see [1] for best fit values of the parameters) already implies physics beyond SM. Neutrinos in that sense seem to be messengers carrying vital information about the structure of physics beyond SM. Neutrinos being electrically neutral opens up the possibility of them being Majorana particles [2]. This Majorana nature however can not be probed in the oscillation experiments that have established non-zero mass of the neutrinos. An unambiguous signature of the Majorana nature of the neutrinos would be a process, $(A, Z) \rightarrow (A, Z + 2) + 2e^-$, called neutrinoless double beta ($0\nu 2\beta$) decay. Such a process violates lepton number by two units [3].

On the experimental side, studies have been carried out on several nuclei ([4]–[10]). Till date however, only one of the experiments [4] (HM) has claimed observation of $0\nu 2\beta$ signal in ^{76}Ge . The claimed half-life is: $T_{1/2}^{0\nu}(^{76}\text{Ge}) = 2.23_{-0.31}^{+0.44} \times 10^{25} \text{ yr}$ at 68% confidence level. By combining the Kamland-Zen and EXO-200 results, both using ^{136}Xe , a lower limit on the half-life $T_{1/2}^{0\nu}(^{136}\text{Xe}) > 3.4 \times 10^{25} \text{ yr}$ can be obtained. This is in conflict with the HM claim. Recently GERDA experiment reported the lower limit on the half-life based on the first phase of the experiment: $T_{1/2}^{0\nu}(^{76}\text{Ge}) > 2.1 \times 10^{25} \text{ yr}$. A combination of all the previous limits leads to a lower limit $T_{1/2}^{0\nu}(^{76}\text{Ge}) > 3.0 \times 10^{25} \text{ yr}$ at 90% confidence level. Both the new GERDA result and the combination are again at variance with the positive claim of HM. Higher statistics in future will shed more light. A different approach can be adopted where comparison of $0\nu 2\beta$ predictions for different nuclei can be made in order to study the sensitivity of theoretical calculations on the nuclear matrix elements (NMEs) used.

Theoretically, it is practically useful to separate the $0\nu 2\beta$ decay amplitude into the long-range and short-range pieces (for a review of theoretical and experimen-

tal issues and the sources of uncertainties and errors, see [11] and references therein). The long range contribution arises when a light neutrino is exchanged while the short range part gets its name from the fact that the intermediate particles are all very massive and therefore the effective interaction becomes point-like once the heavy degrees of freedom are integrated out. The last piece of input is the non-perturbative NMEs, which are the properly normalized matrix elements of the quark level operators sandwiched between the nucleon states. These NMEs are the largest source of uncertainty and the predictions can vary up to a factor of two or more depending upon the NMEs employed (see [12]).

The distinction between the long range and the short range contributions to $0\nu 2\beta$ amplitude is also natural and appropriate from the point of view of renormalization and evolution under renormalization group equations (RGEs). Till recently, the issue of RG running was ignored in the context of $0\nu 2\beta$ studies. It was shown in [13] that perturbative QCD corrections to the short range part can have an important effect on the $0\nu 2\beta$ rate once RG running is incorporated. This would drastically affect the phenomenology as well. More importantly, when different operators are considered, some having a small strength (Wilson coefficient) at the higher scale, and RG evolved to the relevant scale, they can become dominant, and even lead to large cancellations. These effects can be larger than the uncertainties in NMEs that one may envisage. Since there are a large number of models where the scale of new physics is at the TeV, which is being probed by LHC, there is a very interesting correlation between the LHC signatures in such models and $0\nu 2\beta$ rates. There have been many phenomenological studies in this direction (for an incomplete list see e.g. [14]). In all these studies, the fact that RG evolution should be taken into consideration has been ignored. Only recently it has been shown in the context of specific class of models that once the RG effects are incorporated, the impact on the LHC signatures and studies can be significant [15]. The authors of [16] have considered a set of effective operators and have studied the RG evolution for that set, and find that in some cases the limits can change by almost two orders of magnitudes. This set is not complete as the colour mis-matched operators are not considered. A complete calculation, including the colour mis-matched

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operators, will be presented elsewhere. The lesson from these studies is that the RG effects can significantly affect the short range contributions.

The natural question then becomes: Is the long range contribution also affected by QCD corrections? If so, how significantly? In the present note we address this question in affirmation, as we shall see below. For the present, we shall only concentrate on the light neutrino exchange and the SM left-handed gauge interactions. In this context, the inverse half-life (related to the $0\nu 2\beta$ rate) can be expressed as

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu}(Q, Z) g_A^4 |M^{0\nu}|^2 \frac{|m_{\beta\beta}|^2}{m_e^2} \quad (1)$$

where $G^{0\nu}(Q, Z)$, g_A and $M^{0\nu}$ are the phase-space factor, axial coupling and nuclear matrix element of the process respectively. m_e is the electron mass and $m_{\beta\beta} = \sum_i U_{ei}^2 m_i$ is the effective Majorana mass with U being the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix and m_i being the physical neutrino masses. Uncertainties from nuclear physics reside in $G^{0\nu}(Q, Z)$ and $M^{0\nu}$, and also in the value of g_A chosen. Due to the appearance of neutrino masses m_i in $m_{\beta\beta}$, $0\nu 2\beta$ decay has the potential to discriminate between the hierarchies of the neutrino masses. The sum of the neutrino masses gets very tightly constrained from cosmology, which in turn has implications for the mass hierarchies.

Before discussing the structure and impact of QCD corrections, it is worthwhile to briefly outline the steps and important points in the calculation of the rate at the tree level ie without the QCD corrections (see [11]). $0\nu 2\beta$ amplitude has the following parts: (a) two hadronic currents and two leptonic currents - lepton part involves the light Majorana neutrino propagator; (b) as mentioned above, we assume all the vertices to be the standard $V - A$; (c) virtual neutrino is emitted by one nucleon and absorbed by the other, with average momentum flowing through the neutrino propagator, set by the inter-nucleon separation, $q \sim 100 \text{ MeV} \gg m_i \sim \mathcal{O}(0.1 \text{ eV})$, implying a non-local form which decides the type of NMEs to be employed. The leptonic part

$$\mathcal{M}_{lep} = \gamma_\mu (1 + \gamma_5) \frac{1}{\not{q} - m_i} \gamma_\nu (1 - \gamma_5) \rightarrow \frac{m_i}{q^2 - m_i^2} \gamma_\mu \gamma_\nu (1 - \gamma_5) \quad (2)$$

factors out and is dealt with separately. It turns out that (electron momenta are at most $1 \text{ MeV} \ll q$), to a good approximation, the electrons are emitted in S-wave. For the hadronic part, in the second order perturbation theory, one has

$$\mathcal{M}_{had} \propto \sum_n \frac{\langle N, final | J_\mu | N_n \rangle \langle N_n | J_\nu | N, initial \rangle}{E_n + p_{k,elec}^0 + q_{i,initial}^0 - E_{initial}} \quad (3)$$

where J_α, J_β are the hadronic currents. Since $q \sim 100 \text{ MeV} \gg E_n - E_{initial}$ (excitation energy $\sim 10 \text{ MeV}$), one replaces E_n by an average quantity \bar{E} (with this the sum over the intermediate states can be replaced with unity)

- this is the *Closure Approximation* and along with the *Impulse Approximation* for the hadronic currents, renders the nuclear matrix element calculations easy.

Next consider one loop QCD corrections to the tree level $0\nu 2\beta$ diagrams. Representative diagrams are shown in Fig. 1: box diagram and vertex diagram. Let us con-

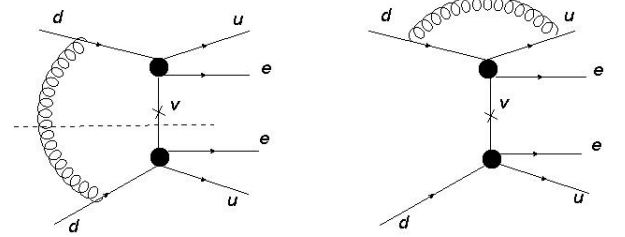


FIG. 1: Representative Feynman diagrams (drawn using the package JaxoDraw [17]) showing one loop QCD corrections: Box (Left) and vertex (Right) diagrams. The thick circles are the standard four Fermi weak interaction vertices while the “cross” on the neutrino line denotes the Majorana nature. The horizontal dashed line denotes possible ‘cut’, leading to an absorptive part.

sider the two type of diagrams one by one and briefly discuss the salient features of both. Before that, it is quite worthwhile to mention an important point that is brought out by the calculation. Due to the loop integrals, one has to deal with complicated functions, typical of such calculations, and as a first step one makes some approximation by neglecting some masses (like quark and electron mass) compared to a larger mass scale (here nucleon mass) ie one makes a series expansion in terms of the small ratio of the masses. Also, the final nucleon momentum squared is expressed as $(1 - \delta)M_n^2$, and δ is then used as an expansion parameter after expressing all four momentum dot products in terms of δ . This is done here as well and the results presented should be looked as first/leading term in such an expansion. This allows to have the leading terms expressed in compact analytic form. For the box diagram, the situation is more involved and interesting. First of all, after making the suitable expansion and retaining the leading terms, the form of the amplitude is local ie point like rather than non-local as expected from the light neutrino exchange. The presence of a hard gluon exchanged between the two quarks in different nucleons allows for the neutrino to go on-shell ie there is an absorptive part. This feature is quite distinctive from the tree level amplitude where this is not possible as $q \gg m_i$. In the intermediate stages, we use gluon mass to regulate the infra-red divergences. At the end of the calculations this parameter is safely set to zero. Here we present the main results of the calculation. The technical details and a systematic numerical study with specific new physics models will be presented in a separate publication [18].

Vertex diagram: it can be decomposed into correction to tree level ($\mathcal{A}^{(1)}$), and a new right handed current con-

tribution (\mathcal{A}^{LR}).

$$\mathcal{A}^{(1)} \sim - \left(\frac{4\alpha_s}{3\pi} \right) \left[\left(\frac{8}{5} + \frac{3i}{5} \right) - 4i \right] \otimes \langle Tree \rangle \quad (4)$$

where we have set the renormalization scale $\mu = M_n$ all through the calculation. This is a large correction to the tree level amplitude.

$$\begin{aligned} \mathcal{A}^{LR} \sim & -\frac{16i\alpha_s}{3} \frac{m_i}{q^2} \left(\frac{m_u m_d}{M_n^2} \right) \\ & \otimes \left(\frac{G_F}{\sqrt{2}} V_{ei} V_{ud}^* \right)^2 \langle J_{V-A}^{had} J_{V+A}^{had} \rangle \mathcal{M}_{lep} \end{aligned} \quad (5)$$

It is worth noting that there was no right handed contribution to start with (only $V - A$ currents were considered) but QCD brings in a small admixture of right handed contribution. This has important phenomenological implications since this QCD generated right handed contribution mimics the new physics contributions typical of left-right symmetric theories. A quick comparison with expectation from left-right symmetric theory with heavy gauge bosons $W_R \sim 3$ TeV shows that this loop induced left-right contribution is at the same level, if not larger.

Box diagram: As mentioned briefly above, box diagram generates a new structure which in the said approximation turns out to be of the local form, though the neutrino exchanged is still a light one. In fact, the box diagram contribution without making such an approximation does not resemble any of the forms that typically appear in $0\nu 2\beta$ calculations, and therefore may require new NMEs which are presently not available. Within the above mentioned approximation, it reads

$$\begin{aligned} \mathcal{A}^{box} \sim & \left(\frac{2\alpha_s m_i}{M_n^2} \right) \left(\frac{2i}{3} + \frac{2}{5} \right) \\ & \otimes \left(\frac{G_F}{\sqrt{2}} V_{ei} V_{ud}^* \right)^2 \langle J_{V-A}^{had} J_{V-A}^{had} \rangle \mathcal{M}_{lep} \end{aligned} \quad (6)$$

This contribution is exactly of the form of a heavy neutrino exchange but the denominator is now mass of the nucleon rather than that of the heavy neutrino which is integrated out in the effective description to get a local point like vertex. Or thinking in terms of light neutrino, the denominator is now M_n^2 instead of q^2 , but the net effect in the end is to involve short range NMEs rather than the conventional long range NMEs for the light neutrino exchange. This would compete with the local heavy neutrino contribution.

We can now assemble various pieces to get an idea about the impact of QCD corrections on the light neutrino exchange contribution. First of all, the vertex corrections leads to a reasonably large correction to the amplitude - almost double. Secod, the right handed contribution is also not suppressed and the fact that the box diagram contribution is of the local form and needs different NMEs than the light neutrino contribution. The

net effect of all this is to effectively enhance the amplitude by a factor 2.5-3 ie the total rate will get enhanced by a factor of $\sim 5-10$. This is a very large correction, and part of it could be the artifact of the approximation made, namely, retaining just the leading term in the expansion with the expansion parameter being the ratio of small masses to the nucleon mass. In fact, it is not possible to handle even the first sub-leading term since this forces one to have very different nuclear physics calculations for the matrix elements. These are not available at the moment and may require a complete rethinking on the nuclear physics side. The essential point is that the structure changes due to loop integrals and one is perhaps thrown away from the closure and/or impulse approximation. This in itself is interesting and merits a detailed investigation, beyond the scope of the present study. Some clue may be possible if one simultaneously studies $\beta\beta$ decay with QCD corrections included since at least the absorptive part of the box diagram can perhaps be recast in a form close to the $\beta\beta$ amplitude by making use of the optical theorem. One can then try to relate some matrix elements. This is quite ambitious at this stage but worth investigating.

At this stage, let us be somewhat practical and look for the implications of such large corrections, assuming the validity of the results obtained. Since the rate is enhanced by a large factor (called K), which we take as $K = 8$, like a mid-value from the given range. This is much larger than the uncertainties in NMEs. A direct consequence of this large K -factor is the decrease of the half-life $T_{1/2}^{0\nu}$ by the same amount. Recalling that different sets of NMEs differ by a factor of 2 or so, and the fact that any uncertainty in g_A will have a large effect as it enters as g_A^4 in the rate, one sees that the $0\nu 2\beta$ rate increases and therefore the half life decreases by a factor of about 20. This is a very large change. From the studies on $0\nu 2\beta$ at the tree level, it is known that only quasi-degenerate neutrinos tend to saturate the experimental limits. However, cosmological constraints on the sum of neutrino masses already disfavour this. The inverted hierarchy predictions for the half-life are typically less than an order of magnitude away from the experimental limits (see eg Dev et al (arXiv:1305.0056 [hep-ph]) in [14]). The impact of QCD corrections would be to disfavour a large part of the inverted hierarchy predictions as well. For the normal hierarchy, the allowed range for the lightest neutrino mass becomes: $m_{lightest} \lesssim 0.03$ eV, while without the QCD corrections it is about $m_{lightest} \lesssim 0.08$ eV. We therefore see that QCD corrected rates can altogether alter all of phenomenology and the inferences we draw. The impact will be very significant and different on LHC related phenomenology.

Let us summarise the main points that have emerged from QCD correcting the $0\nu 2\beta$ amplitude at one loop:

- (i) There is a large enhancement of the tree level amplitude, $\sim \mathcal{O}(2)$
- (ii) Like in left-right symmetric models, there is a left-right mixed contribution which is generated. This con-

tribution can be as large, if not larger than, the one in left-right symmetric theories, and does not depend on the left-right mixing parameter and heavy gauge boson mass (iii) One piece, originating from box diagrams, has a local structure and therefore requires short range NMEs in contrast to the expected long range NMEs.

The net effect of all these is to drastically lower the half-life predictions - almost by an order of magnitude. Not only is it then possible to (almost) saturate the experimental limits simply by the light neutrino contribution, this will have far reaching implications for phenomenology. Let us also recall that large QCD effects have shown to be present even for the short range contributions. In the context of popular scenarios, it was shown that [13] there is a tendency of large cancelations between various short range contributions. Combined with large positive corrections to the light neutrino contribution, this on the

whole appears fully consistent. Moreover, it (long range plus short range contributions to $0\nu 2\beta$ rate) opens up the possibility that the limits from $0\nu 2\beta$ do not come in severe conflict with LHC signatures and searches. Before closing, we would again like to stress that the results have been obtained within some approximations, which allow a tractable calculation at present, and it is important to try to go beyond those approximations. This however seems to require some new nuclear physics calculations. A detailed study, including numerics, with both the long range and short range contributions along with their QCD corrections will be presented in a separate study [18]. It is important not to ignore the QCD corrections to $0\nu 2\beta$, but to incorporate them when comparing with experimental limits and in phenomenological studies.

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